Valid Deduction Worksheet Math 180

Name: Solutions Section: ____________

1. Write down the definition of a *valid deduction*

An argument is <u>valid</u>
if in every universe (row) where all assumptions true,
the conclusion is <u>ALSO</u> true.

2. Use the **definition** of "validity" to prove that the following deduction is valid.

A1 $P \to Q$
A2 $Q \to R$
C $P \to R$

This is another "basic valid deduction," called *Transitivity.* You should **memorize it!**

| P | Q | R | $P \to Q$ (A1) | $Q \to R$ (A2) | $P \to R$ (C) |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | F | F |
| T | F | T | F | T | T |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | T | F | T | F | T |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

The conclusion <u>is</u> true in all rows where A1 & A2 are both true

It follows that the argument <u>IS</u> valid

3. There is one last "basic valid deduction."

A1 $P \vee Q$
A2 $P \to R$
A3 $Q \to S$
$R \vee S$

This argument is called *Dilemma.* You should **memorize** this one too!

Write down an informal argument (in words) explaining why this is true.
(**Hint:** in a "dilemma," you are in one of two cases. Look at what happens in each case.)

we know that either P is true, or Q is true.
There are two cases

<u>Case 1</u>:
Suppose P
By A2 & MP, we get R
So we clearly have R V S

<u>Case 2</u>
Suppose Q.
By A3 & MP, we get S
So clearly we have R V S

In either case, R V S must be true.

The conclusion <u>must</u> be true when all assumptions are true.

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*Some solutions*: Look at each argument, use letters to represent the statements, then determine if the argument is valid or not.

1) Let N represent "we go north" and S represent "we ski".

Then If we go north then we ski. becomes $N \to S$

We go north. $N$

Wek Ski. $S$

$$\begin{array}{l} N \to S \\ N \\ \hline S \end{array}$$

which is exactly the form of MP, so it *is* a valid argument.

Similarly,

2) Let S represent "I sleep" and D represent "I study".

Then If I do not sleep then I do not study. becomes $\neg S \to \neg D$

I do not sleep. $\neg S$

I do not study. $\neg D$

$$\begin{array}{l} \neg S \to \neg D \\ \neg S \\ \hline \neg D \end{array}$$

which is again exactly the form of MP, since $\neg S$ is one premise and $\neg S \to \neg D$ is the other premise, so by MP we conclude $\neg D$. Thus, this is a valid argument.

3) Not valid: Let L be F and M be T.

4) Valid by MT

5) Valid by DS

6) Consider the argument

$$\begin{array}{l} S \to W \\ \neg W \to \neg H \\ \hline S \to H \end{array}.$$

Note that witnessing that this argument is invalid is the same as making the implication

$$((S \to W) \wedge (\neg W \to \neg H)) \to (S \to H)$$

false. To do this, we try to make the hypotheses true while making the conclusion false. The only way that the conclusion, $S \to H$, could be false would be if $S$ were true and $H$ were false.

Now to make the premise, $S \to W$, true, since $S$ must be true (to keep the conclusion false) we make $W$ true. Note that $\neg W \to \neg H$ is equivalent to $H \to W$ by contrapositive. So we can make $H \to W$ true by making H false.

In other words, this truth assignment $\begin{array}{c|c|c} S & H & W \\ \hline T & F & T \end{array}$ makes the conclusion false while keeping the premises true, thus demonstrating that the argument is not valid.

7) Note that it is probably not a good idea to represent statements by T or F. So, "the temperature reaches –22 F" should be represented by TE or TP or TEMP but not by T, because that would create a confusion with T for true.

$$\begin{array}{l} TEMP \to EF \\ G \to \neg EF \\ TEMP \\ \hline \neg G \end{array}$$

Now, $TEMP$ together with $TEMP \to EF$ gives $EF$ by Modus Ponens (MP).

Also, $G \to \neg EF \equiv EF \to \neg G$ by contrapositive.

Finally, $EF$ (obtained above) and $EF \to \neg G$ together give $\neg G$ by MP.

Thus the argument is valid.

8) Valid by dilemma

9) Not valid. We attempt to show this argument is invalid. First, to make the conclusion false we let $S$ be $F$. Then, to make the premise, $B \to S$, true, we must label $B$ as $F$. Working, in this way, since we have $L \to B$ , then $L$ must be $F$ to make the whole statement true. Finally, since $L$ must be false then $P$ must also be false to make $\neg L \to \neg P$ true. Now we have found a truth assignment which makes the conclusion false and all the premises true, so the argument is not valid.

10) This argument can be translated as

$$\begin{array}{c} H \to S \\ H \to B \\ B \to M \\ \hline S \to M \end{array} .$$

We can obtain $H \to M$ by transitivity, but not $S \to M$. So try to show it is not valid by making $S \to M$ false and all the hypotheses true. How could $S \to M$ be false? $S$ would have to be true and $M$ would have to be false. Working on trying to make the premise, $B \to M$, true, since $M$ is false, we must make $B$ false. $B$ false forces $H$ to be false. This truth assignment makes all the premises true and the conclusion false, so the argument is not valid.

11) Not valid. Use C, GJ, M, WH all true.

12)
$$\begin{array}{l} C \to \neg NY \\ FIVE \to NY \\ C \\ \hline \neg FIVE \end{array}$$

Notice that $FIVE \to NY \equiv \neg NY \to \neg FIVE$ by contrapositive.

So, $C$ and $C \to \neg NY$ give us $\neg NY$.
$\neg NY$ and $\neg NY \to \neg FIVE$ give us $\neg FIVE$.
Thus the argument is valid.

Note that there are several ways to show that this argument is valid. Here are two:

1. Contrapositive plus 2 applications of MP MP plus MT
2. Contrapositive plus transitivity plus MP

(13) If x is a koala then x is shy.
If x is shy then x is friendly
______________________________
If x is a koala then x is friendly.

Valid by transitivity.

14) Either my uncle is an art critic or muffins are unhealthy.
If aliens write poetry, then muffins are unhealthy and my uncle is an art critic.
But it is assuredly not true both that muffins are unhealthy and aliens do not write poetry.
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Hence, muffins are unhealthy and my uncle is an art critic.

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This is equivalent to
$$\begin{array}{l} U \vee \neg M \\ A \to (\neg M \wedge U) \\ \neg(\neg M \wedge \neg A) \\ \hline \neg M \wedge U \end{array}$$

There are three ways that the conclusion could be false: (i) both $\neg M$ and U could be false (so M is True and U is False), or (ii) $\neg M$ could be False and U could be True (so M is True and U is True), or (iii) $\neg M$ could be True and U could be False (so M is False and U is False).

Now let's be careful here. What would we need to do to show that the argument is valid? We would have to make sure that it was IMPOSSIBLE to make all the premises true while the conclusion was false. To show that the argument is invalid we would only need to finds one truth assignment that made all the premises true while the conclusion was false. Suppose we tried case (iii) where $M$ False and $U$ False (remember this makes the conclusion false). Since $M$ is False, then $\neg M$ is True, so the first premise is true. Since $M$ is False, then $\neg M$ is True and U is False so the conclusion of the second premise is False. Thus the only way the second premise could be true is if A is False. But then the third premise has $\neg M$ True and $\neg A$ True, so $(\neg M \wedge \neg A)$ is True and thus, $\neg(\neg M \wedge \neg A)$ must be False. Therefore we cannot make all the premises true when the conclusion is False UNDER THIS ASSIGNMENT. Does this make the argument valid? NO. We must show that it is impossible to make the premises True while the conclusion is False under ANY truth value assignment. So we must test the other possibilities. Suppose we try case (i) where $M$ is True and $U$ is False.Here the first premise cannot be True.

So let's try case (ii) where $M$ is True and $U$ is True. Since $U$ is True then the first premise is True. Since $M$ is True then $\neg M$ is False, so the conclusion of the second premise is False Thus the only way the second premise could be True is if $A$ is False. So making $A$ False makes the second premise True. Now in the third premise, since $\neg M$ is False, $(\neg M \wedge \neg A)$ is False and thus, $\neg(\neg M \wedge \neg A)$ must be True. Therefore we have made all three premises True while the conclusion ids False, so the argument is INVALID.

15) Note that the conclusion is an "and" statement.

The trees are tall **and** the land is rich.

When is an "and" statement False?

What does this mean in terms of showing whether the argument is Valid or Invalid?

17)
$$\begin{array}{l} N \to \neg(W \wedge \neg S) \\ \neg(W \wedge \neg S) \vee (\neg I \to C) \\ I \wedge N \\ \hline \neg C \end{array}$$
Invalid